Functions

1. A function, f, divides by 3 then adds 7. Write this symbolically in the form $f(x) =$

2. A function, g, adds 7 then divides by 3. Write this symbolically in the form $g(x) =$

3. $f(x) = 2x - 4$. Write in words what the function, f, does.

4. $g(x) = 5(x + 3)$. Write in words what the function, g, does.

For questions 5 to 16:

$f(x) = 2x - 3$

$g(x) = \frac{x + 4}{3}$

$h(x) = \frac{x}{2} - 5$

5. Find $f(7)$

6. Find x when $g(x) = 4$

7. Find $g(26)$

8. Find x when $h(x) = 19$

9. Find h (11)

10. Find x when f (x) = -13

11. Find f^{-1} (x), giving your answer in terms of x.

12. Find f^{-1} (4)

13. Find g^{-1} (x), giving your answer in terms of x.

14. Find g^{-1} (-2)

15. Find h^{-1} (x), giving your answer in terms of x.

16. Find h^{-1} (2)

17. $k(x) = \frac{1 + x}{3x}$

a) Find $k^{-1}(x)$, giving your answer in terms of x.

b) Find $k^{-1}(2)$

18. $j(x) = \frac{4}{5 + x}$

a) Find $j^{-1}(x)$, giving your answer in terms of x.

b) Find $j^{-1}(10)$

Functions **Answers**

1. A function, f, divides by 3 then adds 7. Write this symbolically in the form f (x) =
   $f(x) = \frac{x}{3} + 7$

2. A function, g, adds 7 then divides by 3. Write this symbolically in the form g (x) =
   $g(x) = \frac{x + 7}{3}$

3. f (x) = 2x - 4 . Write in words what the function, f, does.
   The function multiplies by 2 then subtracts 4.

4. g (x) = 5 (x + 3). Write in words what the function, g, does.
   The function adds 3 then multiplies by 5.

   For questions 5 to 16:

   $f(x) = 2x - 3$

   $g(x) = \frac{x + 4}{3}$

   $h(x) = \frac{x}{2} - 5$

5. Find f (7)
   $f(7) = 2 \times 7 - 3 = 11$

6. Find x when g (x) = 4
   $\frac{x + 4}{3} = 4$

   $x + 4 = 12$

   $x = 8$

7. Find g (26)
   $g(26) = \frac{26 + 4}{3} = 10$

8. Find x when h (x) =19
   $\frac{x}{2} - 5 = 19$

   $\frac{x}{2} = 24$

   $x = 48$

9. Find h (11)
   $h(11) = \frac{11}{2} - 5 = 0.5$

10. Find x when f (x) = -13
    $2x - 3 = -13$

    $2x = -10$

    $x = -5$

11. Find $f^{-1}(x)$, giving your answer in terms of x.
    f multiplies by 2 then subtracts 3. The inverse function adds 3 then divides by 2.
    $f^{-1}(x) = \frac{x + 3}{2}$

12. Find $f^{-1}(4)$

$f^{-1}(4) = \frac{4 + 3}{2} = 3.5$

13. Find $g^{-1}(x)$, giving your answer in terms of x.

g adds 4 then divides by 3. The inverse function multiplies by 3 then subtracts 4.

$g^{-1}(x) = 3x - 4$

14. Find $g^{-1}(-2)$

$g^{-1}(-2) = 3 \times -2 - 4 = -10$

15. Find $h^{-1}(x)$, giving your answer in terms of x.

h divides by 2 then subtracts 5. The inverse function adds 5 then multiplies by 2.

$h^{-1}(x) = 2(x + 5)$

16. Find $h^{-1}(2)$

$h^{-1}(2) = 2(2 + 5) = 14$

17. $k(x) = \frac{1 + x}{3x}$

a) Find $k^{-1}(x)$, giving your answer in terms of x.

$y = \frac{1 + x}{3x}$

$3xy = 1 + x$

$3xy - x = 1$

$x(3y - 1) = 1$

$x = \frac{1}{3y - 1}$

$k^{-1}(x) = \frac{1}{3x - 1}$

b) Find $k^{-1}(2)$

$k^{-1}(2) = \frac{1}{3 \times 2 - 1} = \frac{1}{5}$

18. $j(x) = \frac{4}{5 + x}$

a) Find $j^{-1}(x)$, giving your answer in terms of x.

$y = \frac{4}{5 + x}$

$y(5 + x) = 4$

$5y + yx = 4$

$yx = 4 - 5y$

$x = \frac{4 - 5y}{y}$

$j^{-1}(x) = \frac{4 - 5x}{x}$

b) Find $j^{-1}(10)$

$j^{-1}(10) = \frac{4 - 5 \times 10}{} = -4.6$

Functions Extension

For questions 1 to 4:

$f(x) = 2x - 3$

$g(x) = \frac{x + 4}{3}$

$h(x) = \frac{x}{2} - 5$

1. Find the value of x for which $f(x) = g(x)$

2. Find the value of x for which $h(x) = g(x)$

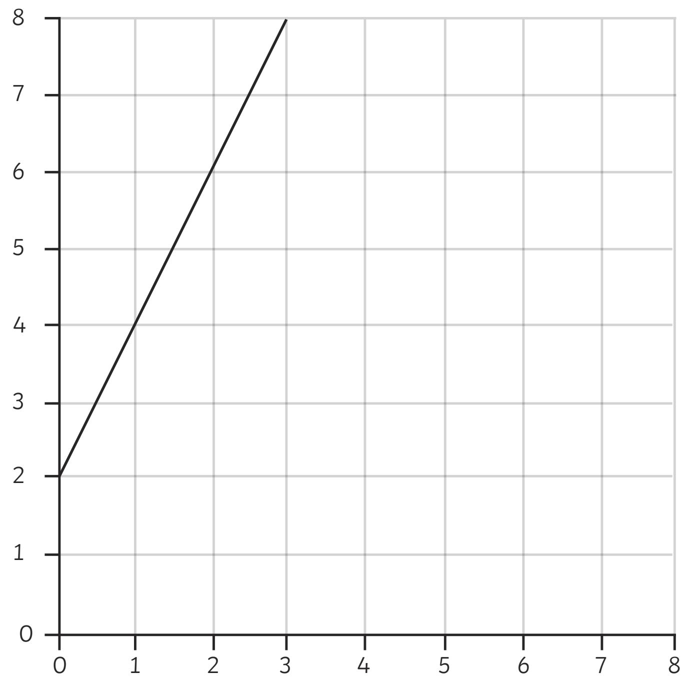
3. Find the value of x for which $f^{-1}(x) = 2h(x)$

4. Find the value of x for which $h^{-1}(x) = f^{-1}(x)$

5. $f(x) = 2x + 3$; when $x > 5$, find the range of possible values of $f(x)$, giving your answer using an inequality.

6. f (x) = x^2; find the range of possible values of f (x), giving your answer using an inequality.

7.



The graph shows y = f (x).

a) Express f (x) in terms of x

b) Find f (2)

b) Find f (7)

8. g (x) = 2x + 5.

a) Express g (2x) in terms of x.

b) Express g (3x + 5) in terms of x.

Functions Extension **Answers**

1. Find the value of x for which f (x) = g(x)

   $2x - 3 = \frac{x + 4}{3}$

   $6x - 9 = x + 4$

   $x = 2.6$

2. Find the value of x for which h (x) = g (x)

   $\frac{x}{2} - 5 = \frac{x + 4}{3}$

   $3x - 30 = 2x + 8$

   $x=38$

3. Find the value of x for which f^{-1} (x) = 2h (x)

   f multiplies by 2 then subtracts 3. The inverse function adds 3 then divides by 2. $f^{-1}(x) = \frac{x + 3}{2}$

   $\frac{x + 3}{2} = 2 \times (\frac{x}{2} - 5)$

   $\frac{x + 3}{2} = x - 10$

   $x + 3 = 2x - 20$

   $x = 23$

4. Find the value of x for which h^{-1} (x) = f^{-1} (x)

   h divides by 2 then subtracts 5. The inverse function adds 5 then multiplies by 2. $h^{-1}(x) = 2(x + 5)$

   $2(x + 5) = \frac{x + 3}{2}$

   $4x + 20 = x + 3$

   $3x = -17$

   $x = \frac{-17}{3}$

5. f (x) = 2x + 3; when x > 5, find the range of possible values of f (x), giving your answer using an inequality.

   $x > 5$

   $2x > 10$

   $2x + 3 > 13$

   $f(x) > 13$

6. f (x) = x^2; find the range of possible values of f (x), giving your answer using an inequality.

   $f(x) \geq 0$, since the result of multiplying any number by itself cannot be negative.

7. a) Express f(x) in terms of x

   $f(x) = 2x + 2$

   b) Find f (2)

   $f(2) = 2 \times 2 + 2 = 6$ or from graph

c) Find f (7)

f (7) = 2 × 7 + 2 = 16

8. g (x) = 2x + 5.

a) Express g (2x) in terms of x.

g (2x) = 2 (2x) + 5 = 4x + 5

b) Express g (3x + 5) in terms of x.

g (3x + 5) = 2 (3x + 5) + 5 = 6x + 15

Algebra
Functions

Learning Objective

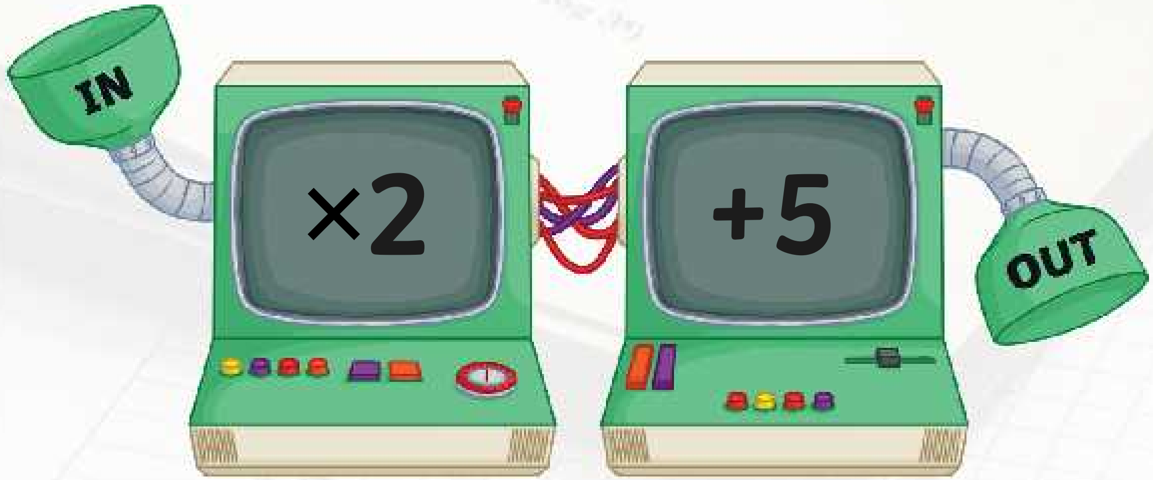
To work with inputs and outputs of given functions, including finding the inverse of a function.

Success Criteria

- To calculate the output of a function given an input, using ( ) notation.
- To calculate the input of a function given an output, using ( ) notation.
- To find the inverse of a given function, using listing and rearranging methods.

Starter: Number Machines

Let's start with a number machine. This number machine multiplies by 2 then adds 5.



Answer the following questions, giving your answers to 3 and 4 in terms of . :

1) If 7 goes into the machine, what number comes out? $7 \times 2 + 5 = 19$

2) If 13 comes out of the machine, what number went in? $(13 - 5) \div 2 = 4$

3) If . goes into the machine, what comes out? 2. + 5

4) If . comes out of the machine, what went in? $\frac{. - 5}{2}$

How Do Functions Work?

A **function** is a mathematical relationship which connects any input to an output. You may have heard them referred to as 'number machines', and that is really what they are; they take a number (the input) and perform some mathematical operations to produce an output.

A function can be written as ( ). stands for function and the in the brackets refers to the input.

In the example in the starter, where the machine multiplied by 2 then added 5, we would write ( ) = 2 + 5. This is saying that the function takes an input, , multiplies it by 2 then adds 5, to give the output.

is the input and ( ) or 2 + 5 is the output.

If we replace the with a number, say 9, we get:
(9) = 2 × 9 + 5 = 23
So when 9 is the input, the output is 2 × 9 + 5 which is 23.

Functions are not always referred to as **f**, they may be **g**, **h** or any other letter, especially if more than one function is used.

How Do Functions Work?

Answer the following:

1) A function, *f*, **divides by 2** then **adds 4**. Write the function, f( ), in terms of .

$(\ ) = \frac{}{2} + 4$

2) $(\ ) = 5(\ + 3)$, find (9)

$(9) = 5(9 + 3) = 60$

3) $h(\ ) = \frac{ - 3}{4}$ Express what this function does, in words.

It subtracts 3 then divides by 4

4) $(\ ) = \frac{}{3} - 5$, find when $(\ ) = 5$

$\frac{}{3} - 5 = 5$

$\frac{}{3} = 10$

$= 30$

Inverse Functions

Every function has an inverse function. If the function is ( ), then its inverse is ⁻ ( ). If the function is ( ), then its inverse is $^{-1}$( ), and so on.

For example

Find the inverse function, ⁻ ( ), of the function ( ) = . +

We are aiming to answer the question: "The number machine multiplies by 10 then adds 7. If **was the output**, write an **expression for the input in terms of** ."

The function **multiplies by 10** then **adds 7**. The inverse function must do **the inverse and in the opposite order: subtract 7** then **divide by 10**.

$^{-1}( ) = \frac{\ - 7}{10}$

Make sure, when you write the inverse function, that you use brackets if necessary, to ensure that the operations happen in the order you need them to, for example,

$( ) = \frac{\ }{5} + 3$

The function divides by 5 then adds 3, so the inverse function must subtract 3 **then** multiply by 5: ( ) = 5(− 3).

Inverse Functions

Answer the following:

1) $(\ ) = 2 \ \ - 9$, find $^{-1}(\ )$

$^{-1}(\ ) = \frac{\ + 9}{2}$

2) $(\ ) = \frac{\ }{5} + 7$, find $^{-1}(\ )$

$^{-1}(\ ) = 5(\ \ - 7)$

3) $h(\ ) = 2(\ \ + 4)$, find $h^{-1}(\ )$

$h^{-1}(\ ) = \frac{\ }{2} - 4$

4) $j(\ ) = \frac{\ - 3}{4}$, find $^{-1}(\ )$

$^{-1}(\ ) = 4 \ \ + 3$

Inverse Functions by Rearranging

So far, we have found the inverse function by:

- **listing** the operations that the function applies to the input;
- finding their **inverses;**
- writing the inverse function using these inverses in the **opposite** order.

However, it is not always possible to do this:

$(\ ) = \frac{4}{\ }$. When we ask what the function does to , the input, it isn't really possible to answer because it is 4 that's being divided by ; the input is happening to 4, rather than something happening to the input.

$(\ ) = \frac{\ }{\ + 1}$. In this function, appears twice, which means that the input is happening to the input.

In either of these situations, we can use another method for finding the inverse function, instead of the list and inverse method.

Inverse Functions by Rearranging

The method is as follows:

- write out the function, with in place of ( ) (or whatever letter denotes the function);
- rearrange the resulting equation to **make . the subject;**
- in the resulting equation, remove the . and replace it with $^{-1}$( ) (or the equivalent inverse function);
- remove all s and replace them with . s.

For example:

( ) = ——— (denominator: . + 1)

Write out your function with in place of ( ).

= ——— (denominator: . + 1)

Rearrange the resulting equation to make . the subject.

(+ 1) = .

. + = .

= . −

= . (1 −)

. = ——— (denominator: 1 -)

In the resulting equation, remove the . and replace it with $^{-1}$( ) and remove all s and replace them with . s.

$^{-1}$( ) = ——— (denominator: 1 - .)

Inverse Functions by Rearranging

$h(.\ ) = \frac{5.\ + 2}{.}$, Find $h^{-1}(.\ )$.

$= \frac{5.\ + 2}{.}$

$. = 5.\ + 2$

$. - 5. = 2$

$.(\ - 5) = 2$

$. = \frac{2}{- 5}$

$h^{-1}(.\ ) = \frac{2}{. - 5}$

Activity Sheet

Work independently through the activity sheet.


Functions

Plenary: Inverse Functions

$(\ ) = \frac{3(2 \ \ + 4)}{2} + 8$

1) Without simplifying, list the operations on the input, in the order that they are applied.

Multiply by 2, add 4, multiply by 3, divide by 2, add 8.

2) Use the list, inverse, opposite method to find $^{-1}(\ )$.

Subtract 8, multiply by 2, divide by 3, subtract 4, divide by 2:

$$^{-1}(\ ) = \frac{\left(\frac{2(\ \ - 8)}{3}\right) - 4}{2}$$

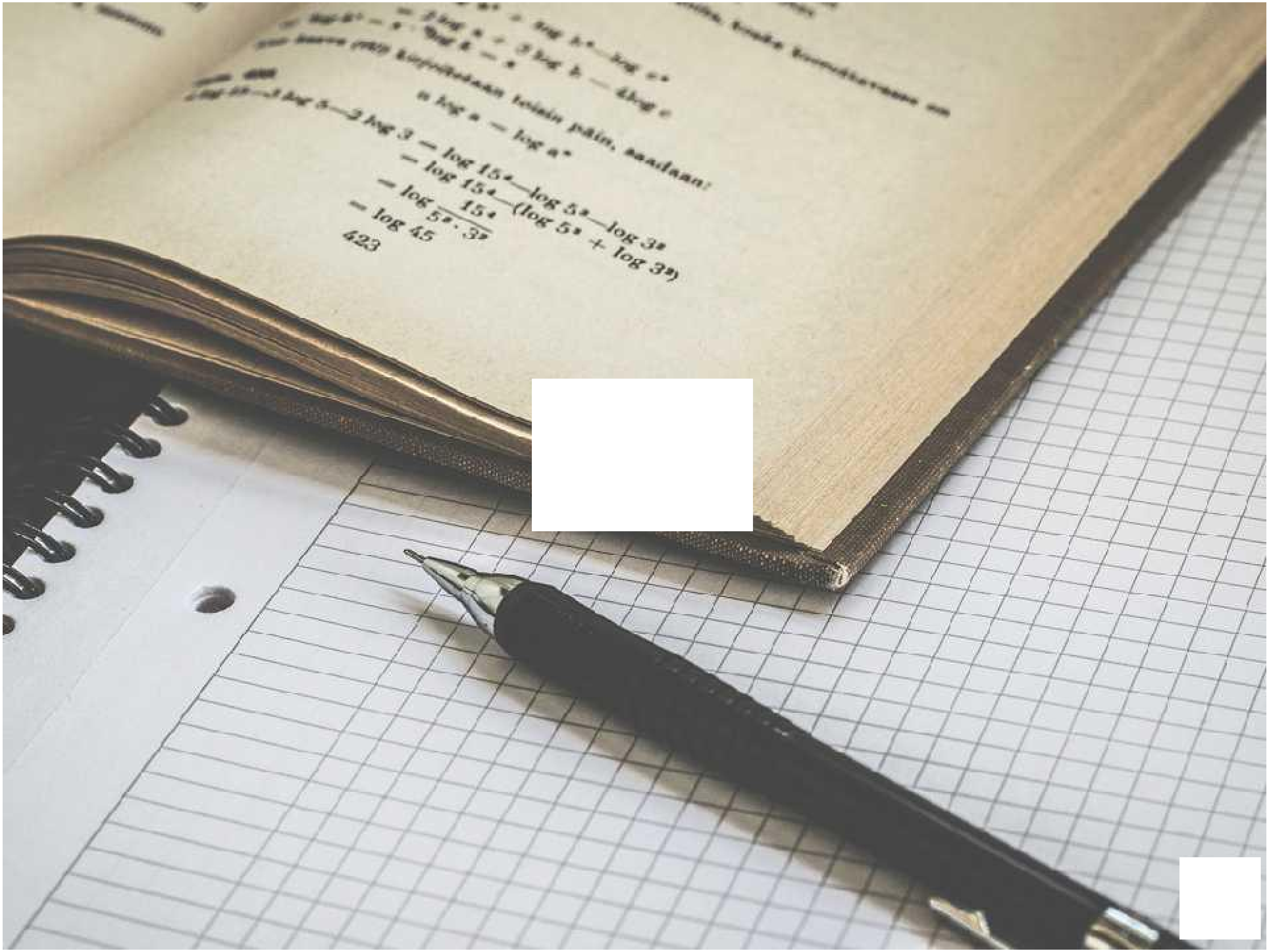
3) Write $(\ )$ in its simplest form:

$(\ ) = 3 \ \ + 14$

4)Use your answer to 2) to write $^{-1}(\ )$ in its simplest form:

$$\frac{\left(\frac{2(\ \ - 8)}{3}\right) - 4}{2} = \frac{2 \ \ - 16 - 12}{6} = \frac{2 \ \ - 28}{6} = \frac{\ \ - 14}{3}$$

(Multiplying numerator and denominator by 3)



Functions
Teaching Ideas

Learning Objective:	To work with inputs and outputs of given functions, including finding the inverse of a function.
Success Criteria:	• To calculate the output of a function given an input, using f (x) notation. • To calculate the input of a function given an output, using f (x) notation. • To find the inverse of a given function, using listing and rearranging methods.
Context:	This is the first lesson on using and manipulating function notation, which is a higher KS4 topic. Another lesson pack covering composite functions is also available. Students should already be proficient and confident with solving equations and rearranging formulae.

Starter

Introduce the concept of a function with a number machine. Ask students to find the output of the number machine when given the input, and find the input when give the output, both numerically and algebraically.

Main Activities

How Do Functions Work?

Define the word function. Reinforce that a function is essentially just a number machine but that students will need to learn some new notation, which looks confusing. Explain how inputs and outputs are referred to symbolically and give some examples.

Inverse Functions

Introduce the notation for inverse functions. Draw parallels with 'I think of a number' questions, e.g. "I think of a number, multiply it by 10 and add 7. How do I obtain the original number?"

Show students how to list the operations of a function in order of their application. Apply the inverses of these operations in the opposite order to create the inverse function. Demonstrate this, introducing and reinforcing the notation.

Allow students to try a few examples for themselves on whiteboards or in pairs.

Inverse Functions by Rearranging

Explain that using the list, inverse, opposite approach does not always work if x appears more than once in a function or if x is the subject rather than the object of an operation. Explain the stages of the rearrangement method for finding the inverse. Give students examples of these.

Activity Sheet

Encourage students to complete the activity sheet independently. A lower ability and higher ability activity sheet are available with answers.

Plenary

Present students with a function containing lots of operations. Ask them to use the listing method to find the inverse, without first simplifying. Now ask students to simplify the function and use the simplified version and the listing method to fine the inverse. Ask them to show that the two inverses are equivalent.